Orbit Dynamics — Resonant Chains and External Companions

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Happy Independence Day!
The Chicago Invasion

Dorian Abbot  Jacobian  Nora Bailey  Jade Checlair
Dan Fabric key  Megan Mansfield  David Martin  Ben Montet
The Chicago Invasion

Dorian Abbot
Jacobean (Jacob Bean)
Nora Bailey
Jade Checlair

Dan Fabric key
Megan Mansfield
David Martin
Ben Montet
Planets are common, especially small ones

Transits/Kepler
(Fulton et al. 2017)
Winn & Fabrycky review. See also Fabrycky et al. (2014)
Multis for which masses are determined – higher mass external planet “catches up” to inner planet by type I migration.
The Resonant Chains

Kepler-223

Kepler-80

TRAPPIST-1

Also transiting:

Kepler-60 (Gozdziewski et al 2016, Jontof-Hutter et al. 2016)
K2-138 (Christiansen et al. 2017)

Doppler: GJ 876 (Rivera et al. 2010, Nelson et al. 2016)
Imaging: HR 8799 (Fabrycky & Murray-Clay 2010, Wang+18, inferred from stability; Gozdziewski & Migaszewski 2014, migration simulation into correct phases)

Broken chains (and collided planets) may form the majority of Kepler sample of multis (Izidoro+17)

https://news.uchicago.edu/story/quartet-exoplanets-locked-complex-dance

https://news.uchicago.edu/story/quartet-exoplanets-locked-complex-dance

(1.518, 1.518, 1.350)

(1.60293 (8:5), 1.67213 (5:3),
1.50622, 1.50939, 1.34174, 1.5192)
Migration into a Resonant Chain

✓ migrated in, perhaps from considerable distance.
✓ first-order resonances

https://www.youtube.com/watch?v=Bi-TFHNVfwY
Transit timing: Kepler-18

Kepler-18b, c, and d: A SYSTEM OF THREE PLANETS CONFIRMED BY TRANSIT TIMING VARIATIONS, LIGHTCURVE VALIDATION, Warm-Spitzer PHOTOMETRY AND RADIAL VELOCITY MEASUREMENTS

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Cochran, Fabrycky et al. 2011
$P = 7.6416 \text{ days}$

$P = 14.8589 \text{ days}$

$P/P = 1.944 \approx 2/1$

Cochran, Fabrycky et al. 2011
“Great Inequality”
frequency:

\[ f_{GI} = \frac{2}{P-1/P} \]

= 0.0037 d\(^{-1}\)

or 270 days
The Great Inequality is observed!

Cochran, Fabrycky et al. 2011
Lithwick+12: Formalized “super-period”; isolating the perturbing terms and m/e degeneracy.

-> Resonant chains naturally have m/e characterized by TTV.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Period (days)</th>
<th>RV Mass (M_{Earth})</th>
<th>TTV Mass (M_{Earth})</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>3.5</td>
<td>12 ± 5</td>
<td>18 ± 9</td>
<td>n/a</td>
</tr>
<tr>
<td>c</td>
<td>7.6</td>
<td>15 ± 5</td>
<td>17.3 ± 1.7</td>
<td>0.00034 ± 0.00014</td>
</tr>
<tr>
<td>d</td>
<td>14.9</td>
<td>28 ± 7</td>
<td>15.8 ± 1.3</td>
<td>0.00045 ± 0.00052</td>
</tr>
</tbody>
</table>
A chain spread: Kepler-80

- $P_f=0.9868\,\text{d};\, P_d=3.0722\,\text{d}$
  \[ P_{ttv}=\frac{1}{|j/P_1-k/P_2|} \]
- $P_e/P_d = 1.518 \quad 192.3\,\text{days}$
- $P_b/P_e = 1.518 \quad 191.9\,\text{days}$
- $P_c/P_b = 1.350 \quad 191.0\,\text{days}$

All these frequency offsets are equal!
Super-duper Period!

Shallue & Vanderburg found 1 small planet, exterior to the 3:2 2\text{ith} planet c, following this pattern!

\[ \phi = p\,\lambda_1 - (q+p)\,\lambda_2 + q\,\lambda_3 \]

\[ \lambda = 360^\circ \left( \frac{1}{4} + \frac{t - T_n}{P} \right) \]

\[ \phi = 360^\circ \left[ -p \frac{T_1}{P_1} + (p + q) \frac{T_2}{P_2} - q \frac{T_3}{P_3} + p \frac{t}{P_1} - (p + q) \frac{t}{P_2} + q \frac{t}{P_3} \right] \]
Tidal spreading of Kepler-80

$10^5$ day eccentricity damping
$10^7$ day semimajor axis damping until migration stops.
Naturally approaches the observed period ratios at same time.

Papaloizou et al. (2010, 2016, 2018), Lithwick & Wu (2012), Batygin & Morbidelli (2013), Lee+13

updating MacDonald et al. 2016
Kepler-80 Laplace resonances

The Laplace angles are exactly where predicted! Modelling the data also confirms small-amplitude libration prediction.

MacDonald et al. 2016
Three-Body Resonance Equilibrium

Delisle (2017), for Kepler-223

For Kepler-223, prediction matches the data!

Key to calculating equilibrium $\phi$ is accounting for non-adjacent resonances, which set equilibrium far from 0 or 180 degrees.

\[ H = -\frac{3}{2} \sum_{i=1}^{3} \frac{n_{i,0}}{\Lambda_{i,0}} \Delta \Lambda_i^2 \]

\[ + \ C_{1,2} \sqrt{D_1} \cos(\sigma_1 - 2\phi_2 - 3\phi_1) \]
\[ + \ C_{2,1} \sqrt{D_2} \cos(\sigma_2 - 2\phi_2 - 3\phi_1) \]
\[ + \ C_{1,3} \sqrt{D_1} \cos(\sigma_1 - 2\phi_2 - \phi_1) \]
\[ + \ C_{3,1} \sqrt{D_3} \cos(\sigma_3 - 2\phi_2 - \phi_1) \]
\[ + \ C_{2,3} \sqrt{D_2} \cos(\sigma_2 - 2\phi_2) + C_{3,2} \sqrt{D_3} \cos(\sigma_3 - 2\phi_2) \]
\[ + \ C_{2,4} \sqrt{D_2} \cos(\sigma_2 - \phi_2) + C_{4,2} \sqrt{D_4} \cos(\sigma_4 - \phi_2) \]
\[ + \ C_{3,4} \sqrt{D_3} \cos(\sigma_3) + C_{4,3} \sqrt{D_4} \cos(\sigma_4), \]

\[ 0 = 2C_{1,2}C_{1,3} \sin(2\phi_1) \]
\[ + 3C_{2,1}C_{2,3} \sin(3\phi_1) + 3C_{2,1}C_{2,4} \sin(3\phi_1 + \phi_2) \]
\[ + C_{3,1}C_{3,2} \sin(\phi_1) + C_{3,1}C_{3,4} \sin(\phi_1 + 2\phi_2), \]

\[ 0 = C_{2,1}C_{2,4} \sin(3\phi_1 + \phi_2) + C_{2,3}C_{2,4} \sin(\phi_2) \]
\[ + 2C_{3,1}C_{3,4} \sin(\phi_1 + 2\phi_2) + 2C_{3,2}C_{3,4} \sin(2\phi_2) \]
\[ + C_{4,2}C_{4,3} \sin(\phi_2). \]
<table>
<thead>
<tr>
<th>Planet</th>
<th>P (d)</th>
<th>P_{i+1}/P_i</th>
<th>n:m</th>
<th>n/P_{i+1}-m/P_i (TTV freq., cycles/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>1.51087081</td>
<td>1.60293</td>
<td>8:5</td>
<td>-0.0060535</td>
</tr>
<tr>
<td>c</td>
<td>2.4218233</td>
<td>1.67213</td>
<td>5:3</td>
<td>-0.0040494</td>
</tr>
<tr>
<td>d</td>
<td>4.049610</td>
<td>1.50622</td>
<td>3:2</td>
<td>-0.0020404</td>
</tr>
<tr>
<td>e</td>
<td>6.099615</td>
<td>1.50939</td>
<td>3:2</td>
<td>-0.0020395</td>
</tr>
<tr>
<td>f</td>
<td>9.206690</td>
<td>1.34174</td>
<td>4:3</td>
<td>-0.0020405</td>
</tr>
<tr>
<td>g</td>
<td>12.35294</td>
<td>1.5192</td>
<td>3:2</td>
<td>-0.002042</td>
</tr>
<tr>
<td>h</td>
<td>18.766</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Super-duper Period is 490 days. *Transit campaign recently derived masses (Grimm et al. 2018).*
Grimm et al. 2018
(updated in Brice’s talk)
TRAPPIST-1 Laplace Resonances
(Luger et al. 2017)

\[
\lambda = 360^\circ \left( \frac{1}{4} + \frac{t - T_n}{P} \right) \quad \phi = 360^\circ \left[ \frac{p T_1}{P_1} + (p+q) \frac{T_2}{P_2} - q \frac{T_3}{P_3} + p \frac{t}{P_1} - (p+q) \frac{t}{P_2} + q \frac{t}{P_3} \right]
\]

<table>
<thead>
<tr>
<th>Planets 1, 2, 3</th>
<th>p</th>
<th>q</th>
<th>( \frac{p}{P_1} - \frac{(p+q)}{P_2} + \frac{q}{P_3} ) (day(^{-1}))</th>
<th>( \phi = p\lambda_1 - (p+q)\lambda_2 + q\lambda_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>b, c, d</td>
<td>2</td>
<td>3</td>
<td>((-4.6, -0.3) \times 10^{-5})</td>
<td>((176^\circ, 178^\circ))</td>
</tr>
<tr>
<td>c, d, e</td>
<td>1</td>
<td>2</td>
<td>((-5.2, +4.5) \times 10^{-5})</td>
<td>((47^\circ, 50^\circ))</td>
</tr>
<tr>
<td>d, e, f</td>
<td>2</td>
<td>3</td>
<td>((-1.9, +1.9) \times 10^{-4})</td>
<td>((-154^\circ, -142^\circ))</td>
</tr>
<tr>
<td>e, f, g</td>
<td>1</td>
<td>2</td>
<td>((-1.4, +1.1) \times 10^{-4})</td>
<td>((-79^\circ, -72^\circ))</td>
</tr>
<tr>
<td>f, g, h</td>
<td>1</td>
<td>1</td>
<td>((-6.0, +0.2) \times 10^{-5})</td>
<td>((176.5^\circ, 177.5^\circ))</td>
</tr>
</tbody>
</table>

The transit times are used to track the \(\phi\) angles of each set of three adjacent planets over the dataset, assuming low eccentricities such that transits occur at a phase angle \(\lambda = 90^\circ\). The ranges of three-body frequency and angle given encompass the changes—most likely librations—seen during the observations.
TRAPPIST-1 migration/damping

Graphs showing the migration and damping over time for the TRAPPIST-1 system, with plots of orbital periods and eccentricities against time. The graphs illustrate the evolution of the planetary system over a period of many thousands of years.
Success! Matches all the period ratios (going beyond Tamayo et al. 2017)

Success! Matches all the correct Laplace angles (amplitudes to be observationally determined)
Tidal Damping coefficients

\[ Q' = Q/k^2 \]

\[ Q' \sim 10^2 \quad \text{terrestrials: Earth: 40, Mars: 470-1000 (Lee et al. 2013)} \]

\[ 10^{4.5} \quad \text{Uranus & Neptune} \]

\[ 10^5-10^6 \quad \text{Saturn & Jupiter} \]

\[ \sim 10^6 \quad \text{moderately eccentric hot Jupiters} \]

Start chains at nominal resonance and spread for the age of the system.

Results:
- Lower limit on \( Q' \), otherwise they would have spread further,
- Upper limit on \( Q' \), otherwise the 3-body libration amplitude would not have damped.
Results:

<table>
<thead>
<tr>
<th>System</th>
<th>R / R_{Earth}</th>
<th>T1d (in days)</th>
<th>M / M_{Earth}</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAPPIST-1</td>
<td>0.3</td>
<td>7</td>
<td>0.8</td>
</tr>
<tr>
<td>Kepler-80</td>
<td>0.8</td>
<td>80</td>
<td>1.5</td>
</tr>
<tr>
<td>Kepler-223</td>
<td>1.5</td>
<td>223</td>
<td>3</td>
</tr>
</tbody>
</table>

- K223b
- K80d
Results:

<table>
<thead>
<tr>
<th>Planet</th>
<th>$M / M_{\text{Earth}}$</th>
<th>$R / R_{\text{Earth}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAPPIST-1</td>
<td>0.3</td>
<td>&gt;2x10^3</td>
</tr>
<tr>
<td>Kepler-80</td>
<td>0.7</td>
<td>tens-hundreds</td>
</tr>
<tr>
<td>Kepler-223</td>
<td>0.8</td>
<td>hundreds</td>
</tr>
</tbody>
</table>

The diagram shows the distribution of planets in terms of their size relative to Earth and their mass relative to the Earth's mass. The green arrow indicates planets in the TRAPPIST system, the blue arrow indicates the Kepler-80 system, and the red arrow indicates the Kepler-223 system. The number of planets per star, with orbital periods less than 100 days, is also plotted.
Exterior massive companions

• The most common type of planets are super-Earths and sub-Neptunes. Seem to form the same population, just lack or have atmospheres. Rather flat and circular (Fabrycky+14, Xie+16, Van Eylen+18).

• Giant planets rarer, but very dynamically active (Juric & Tremaine 08, Chatterjee+08).

• When they coexist (39±7% of the time! – Bryan et al. poster and arxiv:1806.08799, and Zhu & Wu 2018 – and maybe all Jupiters have small companions), what happens?
The case of Kepler-56

Huber et al. 2013
The case of Kepler-56

Huber et al. 2013
The case of Kepler-56

Anticipated by:
Mardling 10 (HAT-P13)
Kaib+11 (55 Cnc?)
Including multiple inner planets torqueing the star:
Boue & Fabrycky 14b →

Simplified by: Lai+18
Applied by: Yee+18 (HAT-P11)

Added stellar spin-down:
Anderson & Lai 18

\[ M_3 \sin i_3 (\text{M}_{\text{Jup}}) \]
\[ e_3 \]

\[ I_0 \]
\[ \theta \]
Mechanism explains the data for Kepler-56 with a probability+duty cycle of 28%.

Sometimes the inner system gets disrupted, which contributes to the “singles” population (Mustill+15, Huang+17).
What about warm Jupiters?

They cannot get to <1AU only by scattering. But their eccentricities rules out pure disk migration.

1) Formation at snow line
   route a:
   2a) disk migration
   3a) eccentricity excitation by external companions
   route b:
   2b) eccentricity excitation by external companions
   3b) tidal dissipation to lower semi-major axis
“Period valley”
Eccentricity-cycling and tidally-decaying at eccentricity peaks

Gillochon+11, Socrates+12, Dawson & Chiang 14, Dong+14, Petrovich & Tremaine 16

Moderate eccentricity-cycling, no tidal dissipation

Anderson & Lai 17 (secular)
Mustill+18 (scattering)
Eccentricity-cycling and tidally-decaying at eccentricity peaks

Gillochon+11, Socrates+12, Dawson & Chiang 14, Dong+14, Petrovich & Tremaine 16

**
For far-out planets, see Baily & Fabrycky poster
**

Moderate eccentricity-cycling, no tidal dissipation

Anderson & Lai 17 (secular)
Mustill+18 (scattering)
Conclusions

- Resonant chains exist! Rare, but useful:
- Simulations calibrate tidal Q for different planet types.
- Inner planetary inclinations and eccentricities can be excited by external companions; outer planets will be a growing theme in the TESS era.